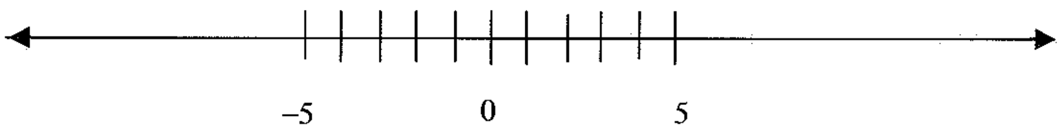
Section 2.1 Positive and Negative Numbers

1. Positive and Negative Numbers on the Number Line: On a straight line, label a convenient point with 0. This is called the origin, and it is usually in the middle of the line. Then label the positive numbers to the right of 0 and the negative numbers to the left of 0. Numbers increase going from left to right. Any number to the left of another number is considered to be smaller that the number to its right.



Example: In each blank, write “smaller than” or “larger than.”

a. –3 is smaller than –2

b. 7 is larger than –10

c. –5 is smaller than 2

2. Inequality Notation: If a and b are any two numbers on the number line, then

$a < b$ is read “a is less than b”

$a > b$ is read “a is greater than b”.

Example: In each blank place the symbol “$<$” or “$>$.”

a. $-7 \underline{\ <\ } -5$

b $-2 \underline{\ <\ } 1$

c. $-10 \underline{\ >\ } -11$

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

3. Absolute Value: The absolute value of a number is its distance from 0 on the number line. We denote the absolute value of a number with vertical lines around the number. Thus the absolute value of –3 is written $|-3|$.

Example: Simplify each of the following.

a. $|-7| = 7$

b. $|8| = 8$

c. $|-6| = 6$

d. $-|-3| = -(3)$
$= -3$

4. Opposite of a Number: Two numbers that are the same distance from 0 are called opposites. The notation for the opposite of a is –a.

Example: Fill in the blanks with the correct answer.

a. The opposite of –5 is __5__.

b. The opposite of 7 is __-7__.

c. The opposite of –10 is __10__.

5. Property of Opposites: If a represents any positive number, then it is true that $-(-a) = a$. Thus, the opposite of a negative number is a positive number.

Example: Simplify.

a. $-(-7) = 7$

b. $-(-8) = 8$

c. $|-7| = 7$

d. $-|-7| = -(7)$
$= -7$

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6. Different Interpretations of "+" and "–" Symbols: The "+" and "–" symbols can be used to indicate addition and subtraction, or to indicate the direction that a number is from 0 on the number line.

Examples:

a. $7 + 6$ The + sign indicates addition.

b. $8 - 3$ The – sign indicates subtraction.

c. -3 The – sign is read "negative" three.

d. $-(-8)$ The first – sign is read the "opposite of" and the second is read "negative" eight. Also, it may be read as "the negative of negative eight."

7. Vocabulary:

Whole numbers: The whole numbers are the set of numbers $\{0, 1, 2, 3, \ldots\}$.

Integers: The integers are the set of numbers $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Example:

a. Name two integers that are not whole numbers.

-6 & -1

b. Is it possible to name two whole numbers that are not integers? Why not? No, all whole numbers are integers as well.

c. Name two numbers that are not integers.

π & $\frac{1}{2}$

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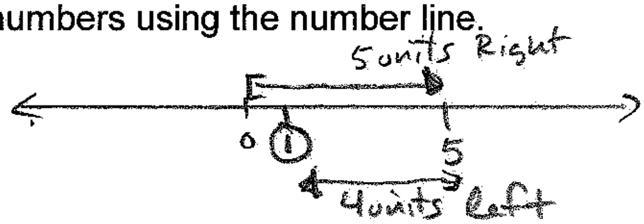
Section 2.2 Addition with Negative Numbers

1. Adding on the Number Line: To add two numbers on the number line:

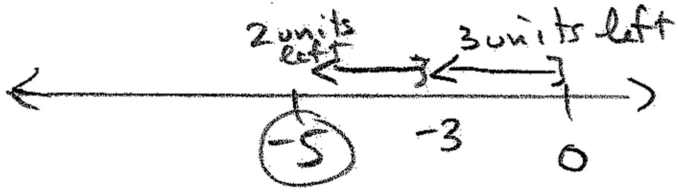
- Start at the origin.
- The first number tells you how far and what direction to move on the number line. Positive means move right, and negative means move left. The absolute value of the number tells you how far to move.
- Starting at the spot where you ended in the step above, use the second number to tell you how far and what direction to move next.
- The ending location is the answer to the problem
-

Example: Add the following numbers using the number line.

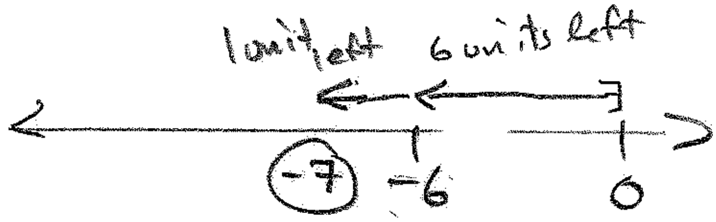
a. $5 + (-4) = 1$



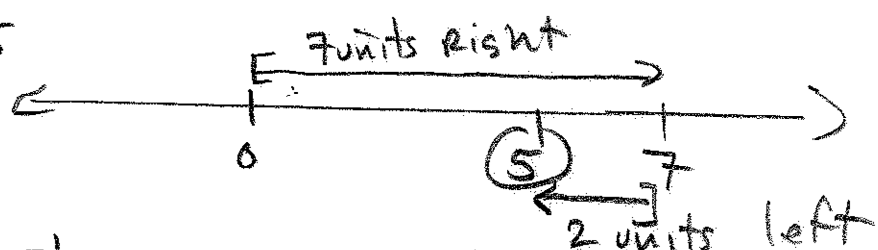
b. $(-3) + (-2) = -5$



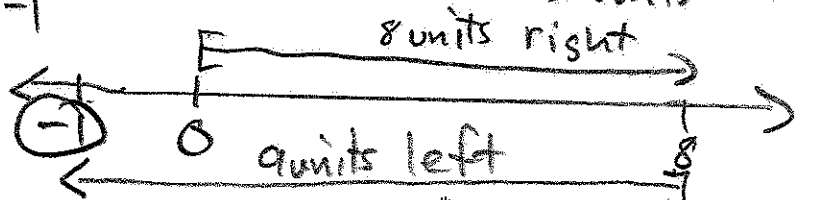
c. $-6 + (-1) = -7$



d. $7 + (-2) = 5$



e. $(8) + (-9) = -1$



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2. Rule for Adding Two Numbers with the Same Signs: To add two numbers that have the same sign:

- Add their absolute values
- Use the common sign.

The sum of two positive numbers is a positive number. The sum of two negative numbers is a negative number.

common sign

Example: Simplify.

a. $(-6) + (-7) = -(|-6| + |-7|)$
$= -(6+7)$
$= -(13)$
$= -13$

b. $(-5) + (-10) = -(|-5| + |-10|)$
$= -(5+10)$
$= -(15)$
$= -15$

c. $(-3) + (-9) = -(|-3| + |-9|)$
$= -(3+9)$
$= -(12)$
$= -12$

3. Rule for Adding Two Numbers with Different Signs: To add two numbers that have different signs:

- Subtract the smaller absolute value from the larger absolute value
- Use the sign of the number that has the larger absolute value.

When you add a positive and a negative number, the sum may be either positive or negative. If the negative number has the larger absolute value, the answer will be negative. If the positive number has the larger absolute value, then the answer will be positive.

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sign of larger absolute value

Example: Simplify.

a. (–7) + 11 $= +(|11| - |-7|)$

$= +[11 - (7)]$

$= +[4]$

$= 4$

b. (–8) + 5 $= -(|-8| - |5|)$

$= -(8 - 5)$

$= -(3)$

$= -3$

c. –7 + 15

$= +(|15| - |-7|)$

$= +[15 - (7)]$

$= +[8] = 8$

Example: Fill in the blank with "same signs" if the problem requires adding two numbers that have the same sign or "different signs" if the problem requires adding two numbers that have different signs.

a. (–8) + 14 different signs

b. (–7) + (–8) Same signs

c. –13 + (–5) same signs

d. –9 + 11 different signs

Example: Simplify

a. (–8) + 14 = 6

b. (–7) + (–8) = –15

c. –13 + (–5) = –18

d. –9 + 11 = 2

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Section 2.3 Subtraction with Negative Numbers

1. Definition of Subtraction: If a and b are any two numbers, then it is true that:

$$a-b=a+(-b)$$

Subtracting a number is the same as adding its opposite.

Example: Write each of the given subtraction problems as an equivalent addition problem using the definition of subtraction.

a. $14-7=14+(-7)$

b. $9-(-4)=9+[-(-4)]=9+4$

c. $-13-5 = -13 + (-5)$

d. $-14-8 = -14 + (-8)$

e. $17-9 = 17 + (-9)$

f. $-15-(-4) = -15 + 4$

g. $-13-(-3) = -13 + (+3)$

h. $17-(-6) = 17 + (+6)$

i. $35-(-4) = 35 + (+4)$

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2. Subtraction with Negative Numbers: To subtract two numbers, rewrite the expression as "addition of the opposite", and then apply the addition rules.

Example: Simplify.

a. $-7-5$

$=-7+(-5)$ change subtraction to addition of the opposite

$=-12$ apply rule for adding numbers that have the same sign

b. $-8-(-5)$

$=-8+[-(-5)]$ change subtraction to addition of the opposite

$=-8+5$ apply rule $-(-a)=a$

$=-3$ apply rule for adding numbers that have different signs

Example: Simplify each of the following.

a. $17-(-10) = 17+(+10)$
$= 27$

b. $-3-10 = -3+(-10)$
$= -13$

c. $4-10 = 4+(-10)$
$= -6$

d. $-15-(-4) = -15+(+4)$
$= -11$

e. $-18-14 = -18+(-14)$
$= -32$

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Section 2.4 Multiplication with Negative Numbers

1. Definition of Multiplication: Multiplication is repeated addition. Thus $3 \bullet 5$ means $5 + 5 + 5$ or $3 + 3 + 3 + 3 + 3$.

Example: Write each of the given multiplication problems as an equivalent addition problem and then simplify.

a. $6 \bullet 4 = 4+4+4+4+4+4$
$= 24$

b. $6(-4) = (-4)+(-4)+(-4)+(-4)+(-4)+(-4)$
$= -24$

c. $7(-3) = (-3)+(-3)+(-3)+(-3)+(-3)+(-3)+(-3)$
$= -21$

d. $(-2)(7) = 7(-2)$
$= (-2)+(-2)+(-2)+(-2)+(-2)+(-2)+(-2)$
$= -14$

2. Rule for Multiplying with Positive and Negative Numbers: To multiply any two numbers:

- Multiply their absolute values.
- The answer is positive if both the original numbers have the same sign.
- The answer is negative if the original numbers have different signs.

In other words,

- A positive number times a positive number gives a positive number.
- A negative number times a negative number gives a positive number.
- A positive number times a negative number gives a negative number.

Example: Simplify each of the following:

a. $17(-10) = -170$

b. $(-8)(-5) = 40$

c. $(-3)(-10) = 30$

d. $4(-10+7) = 4(-3)$
$= -12$

e. $-15(-2)+(-4) = 30 + (-4)$
$= 26$

f. $-3 \bullet 6 + (-7) = -18 + (-7)$
$= -25$

g. $(-5)^2 = (-5)(-5)$
$= 25$

h. $-5^2 = -[5 \cdot 5]$
$= -[25]$
$= -25$

i. $-7^2 = -[7 \cdot 7]$
$= -[49]$
$= -49$

j. $-3(-4)^2 = -3[(-4)(-4)]$
$= -3[16]$
$= -48$

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Section 2.5 Division with Negative Numbers

1. Rule for Dividing with Positive and Negative Numbers: To divide any two numbers

- Divide their absolute values.
- The answer is positive if both the original numbers have the same sign.
- The answer is negative if the original numbers have different signs.

In other words,

- The quotient of two positive numbers is a positive number.
- The quotient of two negative numbers is a positive number.
- The quotient of a positive number and a negative number (or a negative number and a positive number) is a negative number.

Example: Simplify each of the following.

a. $25 \div (-5)$ $= -5$

b. $(-8) \div (-2)$ $= 4$

c. $(-3)(-10) \div (-6)$ $= 30 \div (-6)$
$= -5$

d. $14 \div (-10+8)$ $= 14 \div (-2)$
$= -7$

e. $-15(-2)+(-4)$ $= 30 + (-4)$
$= 26$

f. $-3 \bullet 6 \div (-9)$ $= -18 \div (-9)$
$= 2$

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Section 2.6 Simplifying Algebraic Expressions

1. Simplifying Algebraic Expressions Using the Associative Property: Use the associative property to regroup the multiplication or addition expression so that like factors or terms are together and can be simplified.

Example: Simplify each of the following by first regrouping using the associative property of addition or the associative property of multiplication.

a. $8(4a) = (8\cdot 4)a$
$= 32a$

b. $(-8)(-2a) = [(-8)(-2)]a$
$= 16a$

c. $(-3a)(-10a) = [(-3)(-10)]a\cdot a$
$= 30a^2$

d. $14+(-10+8a) = [14+(-10)]+8a$
$= 4+8a$
or
$= 8a+4$

e. $-15+(2x+5) = (-15+5)+2x$
$= -10+2x$
or
$= 2x-10$

f. $-3+(2x-7) = -3+[2x+(-7)]$
$= [-3+(-7)]+2x$
$= -10+2x$
or
$= 2x-10$

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

2. Simplifying Algebraic Expressions Using the Distributive Property: Recall the distributive property: a(b + c) =ab +ac. We can expand the property to subtraction since we know that subtraction is addition of the opposite. So

Proof: $a(b - c) = a(b + (-c))$
$= ab + a(-c))$
$= ab + (-ac)$
$= ab - ac$

Example: Use the distributive property to simplify.

a. $5(2x + 7) = 5 \cdot 2x + 5 \cdot 7$
$= 10x + 35$

b. $10(3a - 8) = 10 \cdot 3a + 10(-8)$
$= 30a + (-80)$
$= 30a - 80$

c. $-2(3a + 8) = -2 \cdot 3a + (-2) \cdot 8$
$= -6a + (-16)$
$= -6a - 16$

3. Adding or Subtracting Similar Terms: Two terms (addends in an addition expression) are similar if their variable parts are identical. Such terms can be added or subtracted by applying the distributive property. In the answer, the common variable part remains unchanged, but the numbers in front of the variable parts are added or subtracted.

Example: Simplify each of the following.

a. $4x + 3x = (4 + 3)x = 7x$

b. $8a + 10a = (8 + 10) \cdot a$
$= 18a$

c. $3a - 5a = (3 - 5)a = -2a$

d. $11a - 15a = (11 - 15) \cdot a$
$= -4a$

e. $3a + 17 + 5a = 3a + 5a + 17$
$= (3 + 5)a + 17$
$= 8a + 17$

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7[th] ed. by Charles McKeague